1 Definition of Affine Space

An affine space consists of the following: a set A, a vector space $\vec{A}$, and a transitive and free action of the additive group of $\vec{A}$ on the set A. Generally, the set A is used to refer to *the* affine space, and its elements are called *points*, whereas the vector space $\vec{A}$ is said to be *associated* to the affine space, and its elements are referred to as *vectors, translations,* or sometimes *free vectors.*

Recall that a (right) group action of a group G on a set X is a function $\alpha : X \times G \to X$ such that $\alpha(x, e) = e$ and $\alpha(\alpha(x, g), h) = \alpha(x, gh)\ \forall x \in X, \forall g, h \in G$, where $e \in G$ is the identity element. The action is transitive if $\forall x, y \in X$, there exists a $g \in G$ such that $\alpha(x, g) = y$. The action is free if $\alpha(x, g) = x \implies g = e$ for all pairs of elements $x \in X, g \in G$ where g fixes x under the action. In other words, an action is free when no element of X is fixed by a non-identity element of G.

Note that an action being both transitive and free is equivalent to the condition that for every $x \in X$, the mapping $\alpha(x, -)$ is a bijection.